WEEKLY TEST OYJ SOLUTION 16 JULY 2019

MATHEMATICS

31. (d) Let $f(x)=2x^3-24x+107$

At $x=-3,\ f(-3)=2(-3)^3-24(-3)+107=125$

At $x=3,\ f(3)=2(3)^3-24(3)+107=89$

For maxima or minima, $f'(x)=6x^2-24=0$

$\Rightarrow x=2,\ -2$

So at $x=2,\ f(2)=2(2)^3-24(2)+107=75$

at $x=-2,\ f(-2)=2(-2)^3-24(-2)+107=139$

Thus the maximum value of the given function in [– 3, 3] is 139.

32. (c) Given $f(x)=\frac{[(5+x)(2+x)]}{[1+x]}$

$f(x)=1+\frac{4}{1+x}+(5+x)=(6+x)+\frac{4}{(1+x)}$

$\Rightarrow\ f'(x)=1-\frac{4}{(1+x)^2}=0$; $x^2+2x-3=0\Rightarrow x=-3,1$

Now $f''(x)=\frac{8}{(1+x)^3}$, $f''(-3)=-ve$, $f''(1)=+ve$

Hence minimum value at $x=1$

$f(1)=\frac{(5+1)(2+1)}{(1+1)}=\frac{6\times 3}{2}=9$.

33. (c) $a^2x^4+b^2y^4=c^6\ \Rightarrow\ y=\left(\frac{c^6-a^2x^4}{b^2}\right)^{1/4}$

Hence $f(x)=xy=x\left(\frac{c^6-a^2x^4}{b^2}\right)^{1/4}$

$\Rightarrow\ f(x)=\left(\frac{c^6x^4-a^2x^8}{b^2}\right)^{1/4}$

Differentiate $f(x)$ with respect to x, then

$$f'(x)=\frac{1}{4}\left(\frac{c^6x^4-a^2x^8}{b^2}\right)^{-3/4}\left(\frac{4x^3c^6}{b^2}-\frac{8x^7a^2}{b^2}\right)$$

Put $f'(x)=0$, $\frac{4x^3c^6}{b^2}-\frac{8x^7a^2}{b^2}=0$

$\Rightarrow\ x^4=\frac{c^6}{2a^2}\ \Rightarrow\ x=\pm\frac{c^{3/2}}{2^{1/4}\sqrt{a}}$

At $x=\frac{c^{3/2}}{2^{1/4}\sqrt{a}}$ the $f(x)$ will be maximum, so

$$f\left(\frac{c^{3/2}}{2^{1/4}\sqrt{a}}\right)=\left(\frac{c^{12}}{2a^2b^2}-\frac{c^{12}}{4a^2b^2}\right)^{1/4}=\left(\frac{c^{12}}{4a^2b^2}\right)^{1/4}=\frac{c^3}{\sqrt{2ab}}.$$

34. (b) $y = f(x) = -x^3 + 3x^2 + 9x - 27$

The slope of this curve $f'(x) = -3x^2 + 6x + 9$

Let $g(x) = f'(x) = -3x^2 + 6x + 9$

Differentiate with respect to x, $g'(x) = -6x + 6$

Put $g'(x) = 0 \Rightarrow x = 1$

Now, $g''(x) = -6 < 0$ and hence at $x = 1$, $g(x)$ (slope) will have maximum value.

$\therefore [g(1)]_{\text{max.}} = -3 \times 1 + 6 + 9 = 12$.

35. (d) $f(x) = | px - 9 | + r | x |, x \in (-\infty, \infty)$

Where $p > 0$, $q > 0$ and $r > 0$ can assume its minimum value only at one point, if $p = q = r$.

36. (a) Let co-ordinate of R $(x, 0)$

Given $P(1,1)$ and $Q(3,2)$

$$PR + RQ = \sqrt{(x-1)^2 + (0-1)^2} + \sqrt{(x-3)^2 + (0-2)^2}$$

$$= \sqrt{x^2 - 2x + 2} + \sqrt{x^2 - 6x + 13}$$

For minimum value of $PR + RQ$, $\frac{d}{dx}(PR + RQ) = 0$

$$\Rightarrow \frac{d}{dx}(\sqrt{x^2 - 2x + 2}) + \frac{d}{dx}(\sqrt{x^2 - 6x + 13}) = 0$$

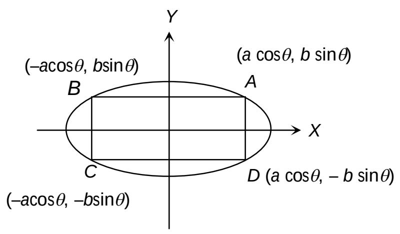
$$\Rightarrow \frac{(x-1)}{\sqrt{x^2 - 2x + 2}} = -\frac{(x-3)}{\sqrt{x^2 - 6x + 13}}$$

Squaring both sides, $\frac{(x-1)^2}{(x^2 - 2x + 2)} = \frac{(x-3)^2}{x^2 - 6x + 13}$

$\Rightarrow 3x^2 - 2x - 5 = 0 \Rightarrow (3x - 5)(x + 1) = 0$, $x = \frac{5}{3}, -1$.

Also $1 < x < 3$. $\therefore R = (5/3, 0)$.

37. (c)



Area of rectangle $ABCD$

$= (2a\cos\theta) . (2b\sin\theta) = 2ab\sin 2\theta$

Hence, area of greatest rectangle is equal to $2ab$, when $\sin 2\theta = 1$.

38. (b) To be increasing $f'(x) = 3x^2 - 27 > 0$

$\Rightarrow x^2 > 9 \Rightarrow |x| > 3$.

39. (b) Since $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing for all real values of x, therefore $f'(x) < 0$ for all x.

$\Rightarrow \sqrt{3}\cos x + \sin x - 2a < 0$ for all x

$\Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x < a$ for all x

$\Rightarrow \sin\left(x + \frac{\pi}{3}\right) < a$ for all $x \Rightarrow a \ge 1, \left[\because \sin\left(x + \frac{\pi}{3}\right) \le 1\right]$.

40. (c) The function $f(x) = x^3$ increases for all x and the function $g(x) = 6x^2 + 15x + 5$ increases, if

$g'(x) > 0 \Rightarrow 12x + 15 > 0 \Rightarrow x > -\frac{5}{4}$.

Thus $f(x)$ and $g(x)$ both increases for $x > -\frac{5}{4}$.

It is given that $f(x)$ increases less rapidly than $g(x)$,

Therefore the function $\phi(x) = f(x) - g(x)$ is decreasing function, which implies that $\phi'(x) < 0$

$\Rightarrow 3x^2 - 12x - 15 < 0 \Rightarrow -1 < x < 5$.

41. (d) If $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$ decreases monotonically for all $x \in R$, then $f'(x) \le 0$ for all $x \in R$

$\Rightarrow 3(a+2)x^2 - 6ax + 9a \le 0$ for all $x \in R$

$\Rightarrow (a+2)x^2 - 2ax + 3a \le 0$ for all $x \in R$

$\Rightarrow a + 2 < 0$ and Discriminant ≤ 0

$\Rightarrow a < -2, -8a^2 - 24a \le 0 \Rightarrow a < -2$ and $a(a+3) \ge 0$

$\Rightarrow a < -2,\ a \le -3$ or $a \ge 0 \Rightarrow a \le -3 \Rightarrow -\infty < a \le -3$.

42. (c) $f'(x) = \dfrac{\sin x - x\cos x}{\sin^2 x} = \dfrac{\cos x(\tan x - x)}{\sin^2 x}$

$0 < x \le 1 \Rightarrow x \in Q_1 \Rightarrow \tan x > x,\ \cos x > 0$

$\therefore f'(x) > 0$ for $0 < x \le 1$

$\therefore f(x)$ is an increasing function.

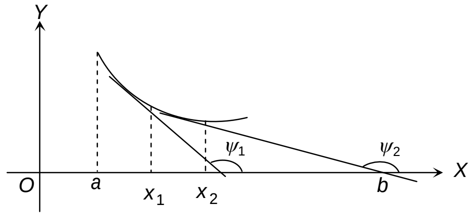
$g'(x) = \dfrac{\tan x - x\sec^2 x}{\tan^2 x} = \dfrac{\sin x\cos x - x}{\sin^2 x} = \dfrac{\sin 2x - 2x}{2\sin^2 x}$

$(\sin 2x - 2x)' = 2\cos 2x - 2 = 2[\cos 2x - 1] < 0$

$\Rightarrow \sin 2x - 2x$ is decreasing $\Rightarrow \sin 2x - 2x < 0$

$\therefore g'(x) < 0 \Rightarrow g(x)$ is decreasing.

43. (d) From the trend of value of $\sin x$ and $\cos x$ we know $\sin x$ and $\cos x$ decrease in $\dfrac{\pi}{2} < x < \pi$. So, the statement S is correct.



The statement R is incorrect which is clear from graph. Clearly $f(x)$ is differentiable in (a, b).

Also, $a < x_1 < x_2 < b$.

But $f'(x_1) = \tan\phi_1 < \tan\phi_2 = f'(x_2)$.

44. (d) Given $f(x) = x^3 + bx^2 + cx + d$

$\therefore f'(x) = 3x^2 + 2bx + c$

Now its discriminant $= 4(b^2 - 3c)$

$\Rightarrow 4(b^2 - c) - 8c < 0$, as $b^2 < c$ and $c > 0$

Therefore, $f'(x) > 0$ for all $x \in R$

Hence f is strictly increasing.

45. (a) $f(x) = 3x^2 - 2x + 1$, $f'(x) = 6x - 2 \ge 0 \Rightarrow x \ge \dfrac{1}{3}$

Option (a) is incorrect. Checking other function similarly we find that they are correctly matched